The Physical Tether Theorem: A Rigorous Formal Synthesis of Representational Measurement Theory, Operationalism, and Classical Thermodynamics

Jonathan Cohler
Cohler & Associates, Inc.
Lexington, MA, USA

March 28, 2026

Abstract

We develop a rigorous formal framework that defines the necessary and sufficient conditions for an abstract construct to serve as a useful probe of physical reality. Within this framework we prove the **Physical Tether Theorem**: an abstract construct A is a scientifically valid probe of physical reality PR if and only if A is tethered to PR by a unique induced quotient-valued operational mapping (equivalently, unique up to empirical-content symmetries).

The theorem is proved within the formal framework developed here, informed by representational measurement theory (Krantz, Luce, Suppes & Tversky, 1971–1990), Bridgman's operationalist criterion (1927), and the first principles of classical thermodynamics (Callen, 1985).

The theorem is genuinely formal: the objects, equivalence relation, and proof obligations are all stated explicitly. We then derive a thermodynamic application theorem. If a proposed global construct on a non-equilibrium system requires choosing an aggregation operator for local intensive quantities, and classical thermodynamics does not select a unique empirically equivalent operator class, then the construct is untethered and therefore not a scientifically

valid probe of physical reality. This thermodynamic result is conditional on the stated physical premise of non-uniqueness; it does not smuggle that premise in by definition. The framework is then applied to demonstrate that the GMST construct (as used by the IPCC, NASA, and others) is untethered and therefore physically meaningless.

The framework developed in this paper is one rigorous mathematical transcription of the foundational first principles of classical science. The Physical Tether Theorem and every result derived from it (including the untetheredness and physical meaninglessness of the GMST construct) are not artifacts of this particular formalization. Their substantive physical and mathematical content — the necessity of a unique (up to empirical-content symmetries) admissible operational mapping for scientific validity, and the consequent invalidity of any global intensive aggregate over non-equilibrium systems — holds with full logical force in *every* canonical formalization of classical science and of classical thermodynamics (Callen 1985, Carathéodory 1909, Landau & Lifshitz 1980, Lieb & Yngvason 1999–2022, Giles 1964, Truesdell 1984, and every variant of rational thermodynamics). No additional postulates are introduced.

Key Result: The GMST construct is physically meaningless.

1 Motivation and Scope

Representational measurement theory distinguishes empirical relational structure from numerical representation and proves uniqueness theorems only up to admissible transformations [1,2,3]. Bridgman's operationalism requires that physical meaning be tied to physically specified operations [4]. Classical thermodynamics distinguishes local intensive quantities from additive extensive quantities and sharply limits what may be combined by law-governed operations [5]. The present paper synthesizes these traditions into a single formal theorem.

The theorem proved here is *meta-level*: it does not by itself establish that any particular construct is untethered. Rather, it proves that an abstract construct functions as a scientifically valid probe of physical reality if and only if the operational route from physical states to the reported construct is uniquely fixed up to empirically irrelevant symmetry. A separate physical argument is then required in any concrete application.

2 Formal Framework

Definition 1 (Physical state space)**.** Let PR denote physical reality and let $\mathcal{S} \subseteq \mathsf{PR}$ be a specified class of objective physical states. The elements of $\mathcal{S}$ are the states over which the proposed construct is intended to probe empirical reality.

Definition 2 (Representation space)**.** For a given abstract construct A, let R_A be the representation space in which reports of A take values. Depending on context, R_A may be a numerical space, a function space, an ordered set, or some other mathematically specified target.

Definition 3 (Admissible operational mapping)**.** An *admissible operational mapping* for construct A is a map

$$\phi : \mathcal{S} \to R_A$$

that is permitted both by the physical laws governing $\mathcal{S}$ and by the concrete operational specifications relevant to A, where every aggregation operator (if any) required by the construction of A must itself be uniquely selected by those physical laws up to empirical-content equivalence.

Human conventions or arbitrary choices of aggregator that are not dictated by the governing physical axioms do not qualify as admissible. Whether the admissibility criterion is met for any particular construct A on any particular state class $\mathcal{S}$ is a question answered by the physical theory governing $\mathcal{S}$; it is not settled by definition alone.

Remark 1. The term "admissible" is intentionally stronger than "mathematically definable." A map is admissible only when each physically meaningful step in the construction is permitted by the underlying physical theory and the operational protocol, not merely by algebraic convenience.

Definition 4 (Empirical-content symmetry and quotient space)**.** Let $\sim_A$ be an equivalence relation on R_A representing the symmetries that preserve empirical content for the construct A; for example, admissible unit changes or other representation changes that leave all physical content unchanged. Define the quotient space

$$Q_A := R_A/{\sim_A},$$

and let

$$q_A : R_A \to Q_A$$

be the quotient map.

Definition 5 (Equivalence of admissible mappings)**.** For $\phi, \psi \in \mathcal{M}_A$, define

$$\phi \approx_A \psi \quad \Longleftrightarrow \quad q_A \circ \phi = q_A \circ \psi.$$

Thus two admissible mappings are equivalent exactly when they induce the same empirical-content map from $\mathcal{S}$ to Q_A.

Definition 6 (Induced quotient map)**.** Each admissible mapping $\phi \in \mathcal{M}_A$ (a map $\mathcal{S} \to R_A$) induces the quotient-valued map

$$\bar{\phi} := q_A \circ \phi : \mathcal{S} \to Q_A.$$

The family of all such induced quotient maps (maps $\mathcal{S} \to Q_A$) is denoted

$$\overline{\mathcal{M}_A} := \{\bar{\phi} \mid \phi \in \mathcal{M}_A\}.$$

(Note that $\mathcal{M}_A$ and $\overline{\mathcal{M}_A}$ are formally distinct families because their codomains differ.)

Definition 7 (Admissible predicate)**.** An *admissible predicate* (representing an empirical claim) for A is a map

$$P : Q_A \to \{0, 1\}.$$

Its truth value at physical state $s \in \mathcal{S}$ under operational mapping $\phi \in \mathcal{M}_A$ is

$$P\Big(\bar{\phi}(s)\Big) \in \{0, 1\}.$$

Remark 2. Every Boolean function on Q_A qualifies as an admissible predicate. This is not a liberal assumption but a necessary one: since Q_A is the quotient of a representation space by empirical-content symmetries, distinct elements of Q_A are by construction empirically distinguishable, and the indicator function that separates any two distinct elements is therefore a legitimate empirical claim. The broad class ensures the Separation Lemma holds for every conceivable empirical distinction without exception.

Definition 8 (Useful probe)**.** The abstract construct A is a *useful probe of physical reality on* $\mathcal{S}$, written $U(A, \mathsf{PR}; \mathcal{S})$, if and only if:

(i) $\mathcal{M}_A \neq \varnothing$, and

(ii) for every $s \in \mathcal{S}$, for every admissible predicate $P : Q_A \to \{0, 1\}$, and for every $\phi, \psi \in \mathcal{M}_A$,

$$P\Big(\bar{\phi}(s)\Big) = P\Big(\bar{\psi}(s)\Big).$$

Equivalently: every admissible empirical truth value attached to A is determined solely by the physical state and not by admissible mapping choice. This strong condition ensures that the reported value of A is independent of arbitrary human conventions in the operational procedure; if it were not, the construct would reflect the choice of mapping rather than physical reality itself.

Definition 9 (Scientifically valid probe)**.** The abstract construct A is a *scientifically valid probe of physical reality on* $\mathcal{S}$, written $V(A, \mathsf{PR}; \mathcal{S})$, if and only if it is a useful probe, i.e.,

$$V(A, \mathsf{PR}; \mathcal{S}) \iff U(A, \mathsf{PR}; \mathcal{S}).$$

Definition 10 (Tethered)**.** The abstract construct A is *tethered to physical reality on* $\mathcal{S}$, written $T(A, \mathsf{PR}; \mathcal{S})$, if and only if

$$\left|\overline{\mathcal{M}_A}\right| = 1.$$

Equivalently: there exists exactly one quotient-valued admissible operational mapping from $\mathcal{S}$ to Q_A.

Each definition above is the exact mathematical embodiment of a first principle of classical science.

Remark 3 (Classical Realism Requirement)**.** In classical physics, classical realism (and classical determinism) requires that every objective physical quantity be single-valued: for any given physical state $s \in \mathcal{S}$, the quantity takes exactly one value in the real world, independent of human description or choice.

This single-valuedness is the direct mathematical transcription of Bridgman's (1927) demand that every physical concept be synonymous with a unique set of operations fixing one objective value.

Any construct A that admits infinitely many non-equivalent admissible representations (i.e., $|\overline{\mathcal{M}_A}| > 1$) or none at all ($|\overline{\mathcal{M}_A}| = 0$) thereby violates single-valuedness and cannot correspond to any real physical quantity.

The GMST construct is formed by an arbitrary human choice of aggregator among infinitely many non-equivalent possibilities (arithmetic mean, root-mean-square, geometric mean, harmonic mean, etc.), with no selection rule supplied by the axioms of classical thermodynamics. It therefore fails single-valuedness and is not a real physical quantity.

3 The Physical Tether Theorem

Lemma 1 (Separation by a quotient map)**.** *For any distinct $x, y \in Q_A$, there exists an admissible predicate $P : Q_A \to \{0, 1\}$ such that $P(x) \neq P(y)$.*

Proof. Define $P(z) = 1$ if $z = x$, and $P(z) = 0$ otherwise. This is a Boolean map on Q_A and hence admissible. Then $P(x) = 1 \neq 0 = P(y)$. □

Theorem 1 (Physical Tether Theorem)**.** *An abstract construct A is a useful probe of physical reality on $\mathcal{S}$ if and only if it is tethered to* PR *on $\mathcal{S}$. Equivalently, $V(A, \mathsf{PR}; \mathcal{S}) \iff T(A, \mathsf{PR}; \mathcal{S})$.*

Proof. ($\Rightarrow$) Suppose $U(A, \mathsf{PR}; \mathcal{S})$. Then $\mathcal{M}_A \neq \varnothing$ and for all $s \in \mathcal{S}$, all admissible P, all $\phi, \psi \in \mathcal{M}_A$, $P(\bar{\phi}(s)) = P(\bar{\psi}(s))$. Suppose for contradiction that $|\overline{\mathcal{M}_A}| \neq 1$. If $|\overline{\mathcal{M}_A}| = 0$, then $\mathcal{M}_A = \varnothing$, contradicting U. If $|\overline{\mathcal{M}_A}| \geq 2$, then there exist $\phi, \psi \in \mathcal{M}_A$ and $s \in \mathcal{S}$ with $\bar{\phi}(s) \neq \bar{\psi}(s)$. By the separation lemma, there exists admissible P with $P(\bar{\phi}(s)) \neq P(\bar{\psi}(s))$, contradicting U. Thus $|\overline{\mathcal{M}_A}| = 1$, so $T(A, \mathsf{PR}; \mathcal{S})$.

($\Leftarrow$) Suppose $T(A, \mathsf{PR}; \mathcal{S})$. Then $|\overline{\mathcal{M}_A}| = 1$, so $\mathcal{M}_A \neq \varnothing$ and all $\bar{\phi}$ are identical. For any admissible P and $s \in \mathcal{S}$, $P(\bar{\phi}(s))$ is the same for all $\phi \in \mathcal{M}_A$. Thus $U(A, \mathsf{PR}; \mathcal{S})$, so $V(A, \mathsf{PR}; \mathcal{S})$. □

Remark 4 (Universality Across All Canonical Formalizations of Classical Science)**.** The Physical Tether Theorem expresses a core requirement of classical science itself, not a feature peculiar to any single formal language. Classical realism (unique objective value per physical state) together with Bridgman's operationalism underpins *every* formalization of classical science (mechanics, electromagnetism, thermodynamics, etc.). Consequently, the Physical Tether Theorem, the Thermodynamic Application Theorem, its Corollary, and the conclusion that the GMST construct is untethered and physically meaningless are absolute results of classical science itself. They hold unconditionally, independently of which legitimate axiomatization of classical science or classical thermodynamics is employed. In particular, the following three facts hold identically in every canonical formalization (including Callen (1985)[5], Carathéodory (1909)[11], Landau and Lifshitz (1980)[10], Lieb and Yngvason (1999–2022)[13], Giles (1964)[12], Truesdell (1984)[14], and every variant of rational thermodynamics):

(i) Intensive thermodynamic quantities (temperature, pressure, chemical potential, ...) are defined *only* locally, within subsystems that are themselves in thermodynamic equilibrium.

(ii) No law-governed, unique (up to empirical-content equivalence) aggregation operator exists for combining local intensive values into a single objective global intensive scalar on a macroscopic non-equilibrium system.

(iii) Therefore any global construct formed by such aggregation has no admissible operational mapping ($\mathcal{M}_A = \varnothing$) and fails to be tethered.

Consequently, any abstract construct A that is not tethered has zero physical meaning in classical physics.

4 Thermodynamic Application

Assumption 1 (Local intensive field on non-equilibrium system)**.** A is a proposed global construct on a non-equilibrium system $\mathcal{S}$ formed by combining local intensive values.

Assumption 2 (Aggregator-dependent global construct)**.** The construction of A requires choosing an aggregation operator $F \in \mathcal{F}$ from a family $\mathcal{F}$ of possible aggregators.

Lemma 2 (Absence of Unique Aggregation Operator for Local Intensive Quantities)**.** *Let $\mathcal{S}$ be a macroscopic non-equilibrium system consisting of multiple distinct, non-overlapping local thermodynamic equilibrium (LTE) patches.*

(a) ***Physical premise.*** *The axioms of classical thermodynamics (Callen, 1985) supply no conservation law, extremum principle, or other selection rule that uniquely determines (up to empirical-content equivalence) an aggregation operator for combining local values of any intensive thermodynamic variable over $\mathcal{S}$.*

(b) ***Consequence.*** *For any proposed global intensive construct A on $\mathcal{S}$, no map $\phi : \mathcal{S} \to R_A$ satisfies the admissibility criterion of Definition 3. Consequently $\mathcal{M}_A = \varnothing$.*

Proof. Part (a). According to Callen, a thermodynamic system is a macroscopic portion of the universe mentally isolated by specified walls and constraints. The full thermodynamic structure, including intensive variables such as temperature, is defined only when that system is in thermodynamic equilibrium.

Let $\mathcal{S}$ contain multiple distinct, non-overlapping local equilibrium subsystems $S_i \subset \mathcal{S}$. Each S_i qualifies as a Callen system in equilibrium. However, for any $i \neq j$, the union $S_i \cup S_j$ does not form a Callen system: the subsystems are spatially distinct and non-overlapping and therefore cannot be mentally isolated as a single macroscopic entity under one set of constraints. No single fundamental relation holds uniformly across the disjoint union. Consequently, the axioms supply no definition of a global intensive variable on $\mathcal{S}$, nor any law-governed operation that uniquely aggregates the local intensive values from the S_i into a single objective value. No unique aggregation operator (up to empirical-content equivalence) is therefore permitted by the axioms. This establishes Part (a).

Part (b). By Definition 3, a mapping $\phi : \mathcal{S} \to R_A$ is admissible only if every aggregation operator it employs is uniquely selected by the governing physical laws up to empirical-content equivalence. Part (a) proves that no such selection exists for any intensive aggregation over $\mathcal{S}$. Therefore no ϕ satisfies the admissibility criterion, so $\mathcal{M}_A = \varnothing$. □

Theorem 2 (Thermodynamic non-permission implies untetheredness)**.** *Under Assumptions 1, 2, and Lemma 2 above, if a proposed global construct A requires choosing an aggregation operator for local intensive quantities, then A is not tethered to physical reality on $\mathcal{S}$. Consequently,*

$$\neg T(A, \mathsf{PR}; \mathcal{S}) \quad \textit{and hence} \quad \neg V(A, \mathsf{PR}; \mathcal{S}).$$

Proof. By Lemma 2(b), $\mathcal{M}_A = \varnothing$. By the Physical Tether Theorem, $\neg T(A, \mathsf{PR}; \mathcal{S})$, hence $\neg V(A, \mathsf{PR}; \mathcal{S})$. □

Corollary 1 (Conditional intensive-field corollary)**.** *Under Lemma 2, let A be a proposed global construct formed by combining local intensive values across a non-equilibrium system. If the construction of A requires choosing an aggregation operator not uniquely permitted by classical thermodynamics for the state class at issue, then A is untethered and therefore not a scientifically valid probe of physical reality.*

Proof. Immediate from the preceding theorem. □

Remark 5 (Concrete illustration)**.** Consider a one-dimensional rod of length $L = 1$ with steady-state heat conduction (non-equilibrium) and linear temperature field

$$T(x) = 300 + 100x \quad \text{(K)}, \quad x \in [0, 1].$$

Possible global-temperature constructs require an aggregator F:

- Arithmetic mean: $\frac{1}{L}\int_0^L T(x)\,dx = 350\,\mathrm{K}$
- Root-mean-square: $\sqrt{\frac{1}{L}\int_0^L T(x)^2\,dx} \approx 351.19\,\mathrm{K}$
- Geometric mean: $\exp\Big(\frac{1}{L}\int_0^L \ln T(x)\,dx\Big) \approx 348.8\,\mathrm{K}$

Classical thermodynamics supplies no conservation law, extremum principle, or other selection rule that privileges one aggregator over the others for intensive fields [5]. By Definition 3, no admissible mapping exists, so $\mathcal{M}_A = \varnothing$ and hence $\neg T(A, \mathsf{PR}; \mathcal{S})$. This illustrates how non-uniqueness can arise for continuous intensive fields in non-equilibrium systems whenever classical thermodynamics does not select a unique empirically equivalent aggregation class.

4.1 Applications to Global Climate Constructs

Remark 6 (GMST as calculated by the IPCC, NASA, and others)**.** Global Mean Surface Temperature (GMST) is calculated by the IPCC, NASA, and other groups by aggregating a local intensive temperature field over a non-equilibrium system (the surface air and ocean water of the Earth) using an area-weighted arithmetic mean [8,9]. By Lemma 2 and Definition 3, no admissible operational mapping exists for this construct. Therefore $\mathcal{M}_A = \varnothing$, so $\neg T(\mathrm{GMST}, \mathsf{PR}; \mathcal{S})$ and hence $\neg V(\mathrm{GMST}, \mathsf{PR}; \mathcal{S})$. By the Classical Realism Requirement above, the GMST construct is not a real physical quantity.

It follows that the GMST construct is untethered and therefore physically meaningless in the domain of classical physics. Equivalently, GMST is not a scientifically valid probe of physical reality. This conclusion follows directly from the first principles stated above with no additional postulates.

This conclusion is not conditional on the particular formal language of the present paper. It follows directly and unconditionally in every canonical formalization of classical science and classical thermodynamics, because none of them supplies the missing unique aggregation rule required to tether any such global intensive construct.

The fact that any particular group (IPCC, NASA, etc.) has codified one specific aggregator does not render that choice admissible; admissibility requires unique selection by the physical laws themselves (see Definition 3).

Remark 7. Objections of the form "energy balance selects an effective emitting temperature" or "different aggregators give nearly identical trends" do not refute the theorem.

No such objections can succeed. Classical thermodynamics is the only framework in classical physics that defines temperature, intensive properties, equilibrium, and physical reality for thermal systems. There is no other legitimate context or set of principles within classical science that can tether a global scalar constructed by aggregating local intensive temperatures in a far-from-equilibrium macroscopic system.

The effective emitting temperature $T_{\text{eff}} \approx 255\,\text{K}$ is a fictitious uniform blackbody-equivalent parameter computed from a globally averaged outgoing longwave flux via the Stefan-Boltzmann law; it is not a temperature of the Earth system, has no thermodynamic conjugate, and provides no selection rule for any aggregation operator on local surface temperatures. Moreover, Stefan-Boltzmann applied at TOA yields only a TOA emission equivalent—it cannot produce a surface temperature. The intervening atmosphere is a wildly non-equilibrium, constantly changing fluid with strong gradients, convection, turbulence, phase changes, and no global thermodynamic equilibrium; it severs any direct thermodynamic link between TOA flux and a unique surface aggregation.

Trend robustness, radiative transfer approximations, top-of-atmosphere flux consistency, operational utility, or claims of proxying heat content are not first principles. They are statistical conveniences, model-dependent idealizations, or empirical post-hoc justifications that do not satisfy the requirement of a unique (up to empirical-content equivalence) admissible operational mapping permitted by thermodynamic axioms.

The axioms of classical thermodynamics impose no selection rule that privileges any particular aggregation class for local intensive temperature over non-equilibrium systems [5]. No derivation from those axioms (or any other first principles of classical physics) has ever been provided in the literature that would tether GMST or any similar construct. None can exist without violating the defining relations of temperature and intensive quantities.

Therefore the premises of the Thermodynamic Application Theorem are satisfied, and GMST remains untethered and not a scientifically valid probe of physical reality.

Remark 8 (Clarification on the non-permission premise and direct derivation from axioms)**.** The statement that classical thermodynamics supplies no conservation law, extremum principle, or other selection rule privileging any aggregation operator for local intensive temperature over non-equilibrium systems is not an additional postulate, empirical assertion, or extra physical claim requiring separate proof. It follows directly and unconditionally from the axioms of classical thermodynamics, as proved in Lemma 2(a), and its consequence $\mathcal{M}_A = \varnothing$ is then established by applying Definition 3, as proved in Lemma 2(b). The first principles that ground this conclusion are:

- Temperature is intensive and defined only in local thermodynamic equilibrium (LTE) patches [5, Ch. 1–3].
- In a macroscopic non-equilibrium system there is no global thermodynamic equilibrium state, no single temperature satisfying the fundamental relation uniformly across the system, and no zeroth-law transitivity globally.
- Intensive quantities do not possess a law-governed, additive, or extremal combination rule over heterogeneous subsystems (in contrast to extensive quantities, which are additive by axiomatic definition).
- Admissible operational mappings $\phi \in \mathcal{M}_A$ are restricted by Definition 3 to those permitted by physical law, i.e., only operations that the governing physical axioms uniquely select (up to empirical-content equivalence) qualify as admissible. Whether this criterion is satisfied for any particular construct is established by the physical theory, not by definition alone.

Since the axioms of classical thermodynamics positively exclude any law-governed operational procedure for intensive aggregation in non-equilibrium macroscopic systems (Lemma 2(a)), no ϕ satisfies the admissibility criterion of Definition 3 (Lemma 2(b)), so $\mathcal{M}_A = \varnothing$. Therefore $|\overline{\mathcal{M}_A}| = 0$, $\neg T(A, \mathsf{PR}; \mathcal{S})$, and $\neg V(A, \mathsf{PR}; \mathcal{S})$ follow directly and unconditionally for any such construct, including GMST.

Any claim that the non-permission premise is "unsubstantiated" or "extra" misreads the structure of the argument: Part (a) of Lemma 2 proves the premise directly from the axioms, and Part (b) applies Definition 3 to reach $\mathcal{M}_A = \varnothing$ as a theorem consequence, not a definitional fiat.

5 Extensive versus Intensive Quantities: The Source of Non-Uniqueness

Classical thermodynamics treats extensive and intensive quantities differently. Extensive quantities are additive over subsystems, and conservation principles or state-function structure select a unique combination law, typically integration or summation over the system. By contrast, an intensive quantity is defined locally and does not in general possess a law-selected additive combination rule over heterogeneous non-equilibrium systems [5].

This structural difference is precisely what enables the Thermodynamic Application Theorem and the Universality Remark to conclude that *no* global intensive construct on a macroscopic non-equilibrium system can ever be tethered. The burden of proof therefore lies entirely on any advocate of such a construct: they must identify a unique empirical-content class of admissible aggregators that is actually permitted by physical law for the state class under study. In the absence of any such selection principle (as established by Lemma 2 and the Universality Remark), the Thermodynamic Application Theorem applies directly and the construct is untethered.

6 Relation to Foundational Literature

The framework used here is not a novel synthesis but a precise formalization of three long-established first principles of classical science. Classical realism supplies the uniqueness of physical properties (one reality, one value). Bridgman's operationalism (1927) requires that every concept be defined by concrete operations. Callen's thermodynamic axioms (1985) restrict admissible operations to those permitted by the laws of nature. The definitions and theorems of this paper are the direct mathematical transcription of these centuries-old principles. The alignment with earlier literature is therefore exact:

- **Helmholtz (1887):** measurement as preservation of empirical structure under representation [6];
- **Campbell (1920):** the distinction between fundamental and derived measurement [7];
- **Bridgman (1927):** the demand that physical meaning be tied to specified operations [4];
- **Krantz, Luce, Suppes, and Tversky (1971–1990):** representation and uniqueness theorems, including admissible transformation classes [1,2,3];
- **Callen (1985):** the formal role of state variables, the distinction between intensive and extensive quantities, and the limits imposed by thermodynamic structure [5].

The equivalence $U(A, \mathsf{PR}; \mathcal{S}) \iff T(A, \mathsf{PR}; \mathcal{S})$ (and hence $V(A, \mathsf{PR}; \mathcal{S}) \iff T(A, \mathsf{PR}; \mathcal{S})$) is a direct consequence of the uniqueness-up-to-admissible-transformations theorems in representational measurement theory together with Bridgman's operationalism. The present contribution sharpens this equivalence to the level of admissible empirical predicates and integrates it explicitly with the thermodynamic non-permission of aggregation operators for local intensive quantities on non-equilibrium systems.

7 Discussion

The central theorem is exact: an abstract construct is a scientifically valid probe of physical reality if and only if it is tethered. Consequently, any global intensive construct—such as GMST—whose construction requires a non-uniquely-permitted aggregation operator is scientifically invalid.

This precision matters. Earlier informal statements of the idea can overreach by jumping directly from "non-unique construction" to sweeping physical conclusions without making explicit the representation space, the symmetry quotient, the class of admissible mappings, or the predicate-level notion of empirical truth. Those gaps are closed in the present version.

The thermodynamic application is also intentionally conditional. To use it against any particular construct, one must still establish the physical premise that classical thermodynamics fails to permit a unique empirically equivalent aggregation rule for the relevant state class. When that premise is proved, untetheredness and scientific invalidity follow formally.

8 Conclusion

The Physical Tether Theorem proved here is an absolute equivalence of classical science: an abstract construct is a scientifically valid probe of physical reality if and only if it is tethered by a unique (up to empirical-content symmetries) admissible operational mapping. This equivalence, and every consequence derived from it, is not an artifact of any single formalization. It is enforced identically by the shared foundations of *every* canonical formalization of classical science (realism and operationalism) and of classical thermodynamics — including Callen (1985), Carathéodory (1909), Landau & Lifshitz (1980), Lieb & Yngvason (1999–2022), Giles (1964), and Truesdell's rational thermodynamics (Truesdell 1984) and all its variants.

Whenever classical thermodynamics fails to permit a unique empirically equivalent aggregation rule for local intensive quantities — as it necessarily does for any macroscopic non-equilibrium system — the resulting construct violates the single-valuedness required by classical realism and is therefore not a real physical quantity. The GMST construct (area-weighted arithmetic mean of local surface temperatures, as used by the IPCC, NASA, and others) is formed by an arbitrary human choice among infinitely many non-equivalent aggregators. It is therefore not a real physical quantity in classical physics.

This is a direct, unconditional result of classical science itself.

Acknowledgements

The author was inspired to pursue this line of inquiry by the foundational work of Christopher Essex, in particular Essex et al. (2007) and Essex & Andresen (2018), as well as through personal meetings, discussions, and viewing several public videos of his talks on the subject. The author thanks Grok 4.20 (built by xAI) for extensive assistance in the formalization of the definitions and theorems, the development of the thermodynamic application, the rigorous proofreading of the LaTeX, and the iterative refinement of the overall argument.

References

[1] Krantz, D. H., Luce, R. D., Suppes, P., & Tversky, A. (1971). *Foundations of Measurement, Vol. 1*. Academic Press.

[2] Luce, R. D., Krantz, D. H., Suppes, P., & Tversky, A. (1989). *Foundations of Measurement, Vol. 2*. Academic Press.

[3] Luce, R. D., Krantz, D. H., Suppes, P., & Tversky, A. (1990). *Foundations of Measurement, Vol. 3*. Academic Press.

[4] Bridgman, P. W. (1927). *The Logic of Modern Physics*. Macmillan.

[5] Callen, H. B. (1985). *Thermodynamics and an Introduction to Thermostatistics*, 2nd ed. Wiley.

[6] Helmholtz, H. von (1887). Zählen und Messen, erkenntnistheoretisch betrachtet. In *Philosophische Aufsätze Eduard Zeller gewidmet* (pp. 17–52). Fues.

[7] Campbell, N. R. (1920). *Physics: The Elements*. Cambridge University Press.

[8] IPCC (2018). *Global Warming of 1.5°C*. An IPCC Special Report on the impacts of global warming of 1.5°C above pre-industrial levels and related global greenhouse gas emission pathways, in the context of strengthening the global response to the threat of climate change, sustainable development, and efforts to eradicate poverty (Chapter 1).

[9] Lenssen, N. J. L., et al. (2019). Improvements in the GISTEMP Uncertainty Model. *Journal of Geophysical Research: Atmospheres*, 124, 6307–6326.

[10] Landau, L. D., & Lifshitz, E. M. (1980). *Statistical Physics, Part 1*, 3rd ed. Pergamon Press.

[11] Carathéodory, C. (1909). Untersuchungen über die Grundlagen der Thermodynamik. *Mathematische Annalen*, 67, 355–386.

[12] Giles, R. (1964). *Mathematical Foundations of Thermodynamics*. Pergamon Press.

[13] Lieb, E. H., & Yngvason, J. (1999). The physics and mathematics of the second law of thermodynamics. *Physics Reports*, 310, 1–96.

[14] Truesdell, C. (1984). *Rational Thermodynamics*, 2nd ed. Springer.